Maxwell's Dynamical Top

Dr Roger C Clark, Department of Mathematical Sciences, Aberdeen University.

Maxwell designed his dynamical top (Fig. 1) specifically *'for exhibiting the phenomena of the motion of a system of invariable form about a fixed point'* [1]. As the motion is exhibited by means of a coloured disc, there can be and has been confusion between his dynamical top and the colour top or teetotum which Maxwell used in his experiments on colour mixing. This is the device he is seen holding in the portrait painted when he was at Trinity College, Cambridge in 1855. A full description of this teetotum (Fig. 2) is given in the manuscript "*On the comparison of colours using a spinning top*" [2]. By contrast, the dynamical top is carefully designed with sufficient adjustments to enable the centre of mass to be positioned precisely at the point of support, the axle to be made a principal axis of inertia and for each of the principal moments to be varied relative to one another. It is first referred to in a letter Maxwell wrote from Trinity to his father in February 1856 [3] where he says '*I took my great top there and spun it with coloured discs attached to it. I have been planning a form of top, which will have more variety of motion, but I am working out the theory so I will wait till I know the necessary dimensions before I settle the plan.*' The 'great top' mentioned here is perhaps the simple top (Fig. 3) (which has no adjustments) that is now in the Cavendish Laboratory, Cambridge along with Maxwell's own dynamical top. [4]

A version of the top was demonstrated to the British Association meeting in Cheltenham in August 1856 [5]. The body of that version was made of wood but it had an iron axle with a steel point. On the axle was an iron nut which served the same purpose as the brass bob on the tops made in Aberdeen. It only had four horizontal bolts for adjusting the centre of mass, instead of six, but, where the later version has three vertical screws, the earlier version had four.

The final brass version was built in Aberdeen by Charles Ramage of the instrument makers Smith and Ramage and was demonstrated to the Royal Society of Edinburgh in April 1857. [1] By December of that year, Ramage was making four tops *'for various seats of learning'* [6]. It is clear from Maxwell's letters that among these were Durham [7] (to R.B. Hayward), Edinburgh [8] (to J.D. Forbes) and Cambridge [9] (to lecturers at Trinity). Maxwell's own version of the top went with him when he moved to Kings College, London [10] and from there to Cambridge. In 1859, a letter accompanying a top sent to P.G. Tait mentions that the Trinity lecturers had '*broke the cup of theirs by bumping it down inconsiderately'.* [9] It also describes a short handle enabling the user *'to spin it and prevent drilling of the finger*'. This was needed because the upper end of the axle is as pointed as the lower end. Maxwell had at one time visualised mounting the top like a gyroscope with the upper end guided by 'a separately balanced swing-frame'. [11]

When he demonstrated the top in Edinburgh in 1857 Maxwell was particularly proud of two innovations. The first was the use of the coloured disc to show the movement of the 'invariable axis' - the angular momentum vector - relative to the axle of the top. The second was his simplification of the mathematics describing this motion by use of the conservation of angular momentum and energy. In his paper he wrote '*The optical contrivance for rendering visible the nature of the rapid motion of the top, and the practical methods of applying the theory of rotation to such an instrument as the one before us, are the grounds on which I bring my instrument and experiments before the Society as my own'.* [12] The following month in a letter to Stokes [13] he wrote '*I have had a dynamical top of brass made at Aberdeen and have been simplifying the theory of the motion of the "invariable axis" (normal to the invariable plane) in the body. The extremity of this axis describes spherical ellipses about the greatest or least principal axes.....*'

Maxwell's method of deriving this motion is still used by modern textbooks on classical mechanics to describe force-free rotation of a rigid body. [14] He showed that the theory can be deduced '*as briefly as possible from two considerations only, - the permanence of the original angular momentum in direction and magnitude and the permanence of the original vis viva.* [15] In modern notation this implies that both

$$L^2 = L_x^{\,2} + L_y^{\,2} + L_y^{\,2}$$

and

$$T = \frac{L_x^{\,2}}{2I_x} + \frac{L_y^{\,2}}{2I_y} + \frac{L_z^{\,2}}{2I_z}$$

are constant where L_x, L_y and L_z are the components of the angular momentum vector **L** in the axes of the body and I_x, I_y, I_z are the principal moments of inertia. Hence, Maxwell concluded, the angular momentum vector sweeps out a cone that passes through the intersection of these two surfaces. He goes on to calculate the rate at which **L** describes this path by observing that '*Since the invariable axis is fixed in space its motion relative to the body must be equal and opposite to that of the portion of the body through which it passes*'. [16]

Maxwell describes how to use the coloured disc to set up the axle as one of principal axes of inertia. This is done by watching the changing colour of the centre of motion when the top is spun initially about its axle. '*If the axis about which the top is really revolving, falls within this disc, its position may be ascertained by the colour of the spot at the centre of motion. If the central spot appears red, we know the invariable axis at that instant passes through the red part of the disc*' [17] Adjustment of the vertical screw nearest the red part of the disc can then be made to reduce the wandering of the axis of rotation from the axle.

After being set up the spinning top must be given a sharp knock to set it spinning about an arbitrary axis. Its behaviour is then determined by the relative values of the principal moments of inertia. Maxwell describes each possible case. If the axle is the axis of the greatest principal moment of inertia the colours are seen in the same order as they appear on the disc in the direction of spin of the top. If the axle corresponds to the least moment of inertia the colours are seen in the opposite order. The greater the ratio of these two moments, the faster the colours change while the angular momentum vector describes a small ellipse. Finally, as Maxwell points out, '*If the axle be made the mean axis, the path of the invariable axis will be no longer a closed curve, but an hyperbola, so that it will depart altogether from the neighbourhood of the axle*'.[18]

He concludes his paper by postulating that the same type of precession might be seen in the earth's motion, while admitting that it could be heavily damped. He predicted the period of this precession to be 325.6 solar days. [19] There is a wobble of the earth's axis with a period of about 420 days, the Chandler wobble, that is believed to be due to the force-free precession Maxwell described. However the correspondence between Maxwell's prediction and the Chandler wobble is not simple. [20]

The top spun in the video demonstration was sold in 1885 by Harvey and Peak the instrument makers of Beak Street, London. It was purchased at that time for six guineas by Professor Charles Niven for Aberdeen University. [21]

Acknowledgements

I am grateful to Mr Stuart Leadstone for suggesting that a video of Maxwell's Dynamical Top should be made, to Mr John Sangster for his able assistance in producing the video and to Dr John S. Reid and the staff of the Physics Unit of Aberdeen University for their help.

References

Most of the references can be found either in '*The Scientific Papers of James Clerk Maxwell*', edited by W.D. Niven; Vol. 1; Cambridge University Press 1890, (hereafter referred to as 'Scientific Papers', 1) or in '*The Scientific Letters and Papers of James Clerk Maxwell*'; edited by P.M. Harman; Vol. 1, 1846-1862; Cambridge University Press 1990 (hereafter referred to as 'Scientific Letters, 1).

1. *'On a Dynamical Top, for exhibiting the phenomenon of the motion of a system of invariable form about a fixed point, with some suggestions as to the Earth's motion'*. Trans. Roy. Soc. Edin. 21, (1857), 559-70. (Scientific Papers, 1, 248-62).

2. Manuscript on the Comparison of Colours using a Spinning Top. *'Description of the Chromatic Teetotum as constructed by Mr J.M. Bryson, Optician Edinburgh'*. 1855. (Scientific Letters, 1, 284-6); see also *'Experiments on Colour, as perceived by the Eye, with remarks on Colour Blindness'*. Trans. Roy. Soc. Edin. 21, (1855), 275-98. (Scientific Papers, 1, 126-54).

3. Letter to John Clerk Maxwell, 14 February 1856, (Scientific Letters, 1, 384).

4. I am grateful to Dr Peter M. Harman and to the Cavendish Laboratory (private communications) for clarifying this point.

5. *'On an Instrument to illustrate Poinsôt's Theory of Rotation'*. Report of the twenty-sixth meeting of the British Association for the Advancement of Science; held at Cheltenham in August 1856. (Scientific Papers, 1, 246-7).

6. Letter to Lewis Campbell 22 December 1857 (Scientific Letters, 1, 576).

7. Letter to Peter G. Tait 21 November 1857 (Scientific Letters, 1, 558).

8. Letter to James D. Forbes 26 November 1857 (Scientific Letters, 1, 568).

9. Letter to Peter G. Tait 9 March 1859 (Scientific Letters, 1, 604).

10. Letter to Michael Faraday 21 May 1861 (Scientific Letters, 1, 681).

11. Letter to James D. Forbes 30 March 1857 (Scientific Letters, 1, 497).

12. Reference 1 page 249.

13. Letter to George G. Stokes 8 May 1857 (Scientific Letters, 1, 502-4).

14. Herbert Goldstein, '*Classical Mechanics*', 2nd Edition. 1980 Addison-Wesley. pp 208-9

15. Reference 1 page 250.

16. Reference 1 page 253.

17. Reference 1 page 257.

18. Reference 1 page 258.

19. Reference 1 page 260.

20. Reference 14 page 212.

21. Revised Catalogue of Scientific Apparatus; Harvey and Peak, Beak Street, Regent Street, London W; 1885. Entry 696 '*Maxwell's Dynamical Top, for illustrating phenomena of rotation. The top is made with all parts and adjustments correctly to size and weight as described by the late Professor Maxwell; packed in mahogany case...£6 6 0.*' (I am grateful to Dr John S. Reid for bringing this to my attention).

Fig. 1

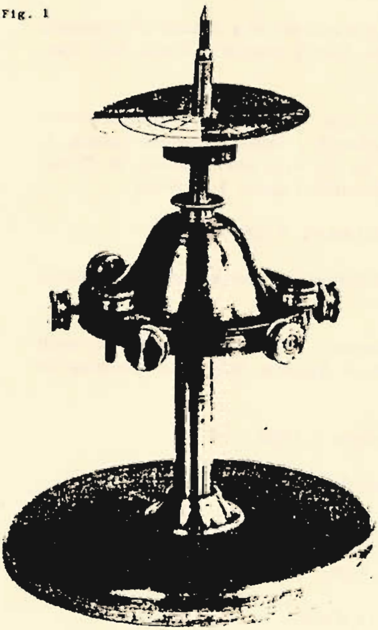

Fig. 2

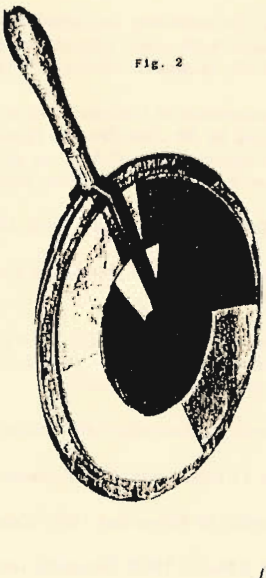

Fig. 3