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Clerk Maxwell and the complex behaviour of an object released from rest in a fluid

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Based (with permission of Cambridge University Press) on my article 'Three Coins in a Fountain' *published in* J. Fluid Mech. *(2013) 720, 1-4.*

Introduction

It is known that James Clerk Maxwell and his wife toured Italy in 1867, including a visit to Rome. History does not record whether, for reasons of tradition or to ensure good luck, Maxwell dropped a coin into the Trevi fountain (where coins worth a total of £1.5 million are now dropped in a year!).

As Maxwell had already published a related article, he would no doubt have observed with a keen eye the manner of the coin's descent. The interesting question is this: if, in a large expanse of fluid, a solid object, of density greater than the fluid, is released from rest, how exactly does it fall?

Maxwell had, in 1853 (at age 22), published a paper entitled *"On a particular case of the descent of a heavy body in a resisting medium"* [1]. In the opening paragraph he wrote:

"Everyone must have observed that when a slip of paper falls through the air, its motion, though undecided and wavering at first, sometimes becomes regular. Its general path is not in the vertical direction, but inclined to it at an angle which remains nearly constant, and its fluttering appearance will be found to be due to a rapid rotation about a horizontal axis. . . ".

Maxwell had himself investigated this phenomenon by dropping a slip of paper of size some two-inches by one-inch. The experiment can be easily repeated; a stairwell provides an ideal setting as some height is needed for Maxwell's regular motion to be established.

Maxwell's discussion of this phenomenon and the physical insight that he displayed was remarkable, given that, in 1853, the effects of the viscosity of fluids (such as air) were then but dimly understood. It was his first venture into fluid mechanics, a field to which he later made a number of seminal contributions which deserve to be better known[2].

Motion of a falling object

The large expanse of fluid may be an ocean, a large bath of water or a large mass of air. The solid object might be a coin, a metal plate, a seed, a sinking ship or an artificial satellite falling through the atmosphere; Maxwell's slip of paper had some flexibility, a complicating feature.

Familiar to most is the descent in air of the winged seed of a sycamore tree. The shape of the seed is responsible for the spinning, sideways, helicopter-like descent that is observed. No doubt its shape has evolved in this way to ensure that the seed makes a soft landing as far from the parent sycamore tree as possible!

There are many different possible modes of motion by which an object may descend through a fluid; determining which mode is adopted in any particular situation is one of the central problems of fluid dynamics and, as such, has been studied intensively over the past 150 years.

Clearly the mode (or modes) will depend on the object's shape, size and density and the density and viscosity of the surrounding fluid. The shape of the body can usually be described by a small number of parameters, e.g. the ratio of length to width for a rectangular plate, or the ratio of thickness to radius for a circular coin.

The application of *Archimedes' Principle* (the up-thrust equals the weight of fluid displaced) gives the object a downward acceleration $g(\rho_s/\rho_f - 1)$, where ρ_s/ρ_f is the ratio of the density of the solid object to that of the fluid. But *Archimedes' Principle* takes no account of the drag (caused by fluid viscosity) as the object descends.

Viscosity, no matter how small, is recognised as being of crucial importance in problems of this kind: without the viscosity of air, the flight of bats, birds, bees and aircraft and the dispersal and soft landing of the sycamore seed would be impossible!

Situation when the object is fixed

When an object, such as a long plate or cylinder or a sphere, is fixed in a uniform stream of fluid, it experiences a drag force associated with the rotational flow (the *'vorticity'*) in the downstream wake. This *vorticity* (created by viscosity) arises in a thin boundary layer on the surface of the object and is then swept into the wake through a physically self-evident, though mathematically complex, *'separation'* mechanism. The famous *'von Karman vortex street'* shed from a fixed cylinder in a moving fluid is perhaps the best-known example of this separation phenomenon.

A similar boundary-layer effect associated with viscosity is present when the object is free to fall.

Much experimental evidence of the mode of descent is available under a wide range of circumstances but, even for the case of a circular disc, the full problem is currently beyond the reach of a purely theoretical analysis and has only recently become amenable to investigation through the use of powerful computational techniques[3, 4, 5]. The challenge is to reproduce, by computation, the experimental observations.

1 Maxwell, J.C. (1853), On a particular case of the descent of a heavy body in a resisting medium. *The Scientific Papers of James Clerk Maxwell,* Vol. 1. (Camb. Univ. Press, 2010), 115–118.
2 Moffatt, H.K. (2013), The fluid dynamics of James Clerk Maxwell. In: *James Clerk Maxwell,* Eds. R. Flood, M. McCartney & A. Whitaker (Oxford Univ.Press).

Modes of descent

For the case of a circular disc, it appears from these computations that there are at least six possible modes of descent, the first four of which had been identified experimentally in previous work (e.g. Field et al. 1997)[6]:

(i) a *straight vertical mode*, realised when (ρ_s/ρ_f -1) is sufficiently large, in which the disc is horizontal and the centre descends on a straight vertical path;

(ii) a *zig-zag mode* in which the plane of the disc rocks back and forth while the centre of the object follows a sinusoidal curve, as shown in Figure 1;

(iii) an *autorotation mode*, for which the disc rotates ('tumbles') about a horizontal axis and drifts on an inclined path (this is apparently the mode discovered by Maxwell with his slip of paper);

(iv) an *intermediate chaotic regime*, being a mixture of periods of (ii) and (iii);

(v) a *hula-hoop mode*, being a zig-zag mode in which the plane-of-fall of the disc precesses about a vertical axis;

(vi) a *helical autorotation mode*, being an autorotation mode in which again the plane-of-fall precesses about a vertical axis.

Modes (v) and (vi) were discovered computationally by Auguste et al. (2013)[4] .

Whichever mode is actually adopted depends on three ratios: of solid to fluid density, of disc thickness to radius, and of viscous drag to inertia; and transitions between modes are possible if any of these ratios are slowly changed.

Figure 1:
The zig-zag mode (Auguste 2010)[3]

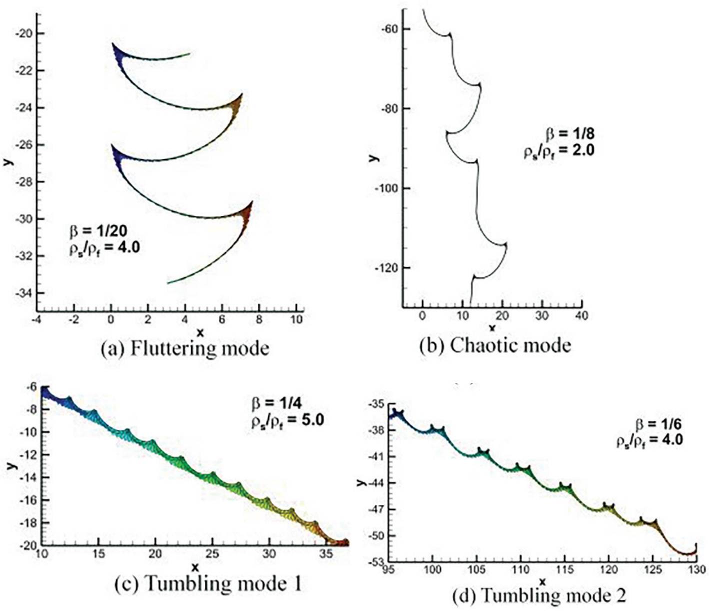

Figure 2:
Various modes of descent of an infinitely long rectangular plate:
Y. Wang (2016)[5] *where β is the ratio of the plate's thickness-to-width.*

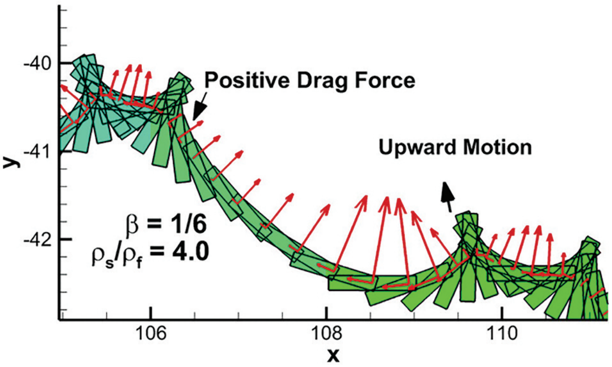

Figure 3:
Fluid forces on the tumbling plate with β = 1/6 and ρs/ρf = 4.0.
Y. Wang (2016)[3] *where β is the ratio of the plate's thickness-to-width.*

The zig-zag mode for a coin is illustrated in Figure 1 (from Auguste, 2010)[3], and, for the case of a long rectangular plate, in Figures 2 and 3 (from Wang et al. 2016)[5] ; videos showing tumbling and zig-zag modes in this case can be found at <**http://dx.doi.org/10.1063/1.4963242**>.

Three coins in a fountain

Figure 4: Three coins: 1 pound (UK); 5 Yen (Japan); and 20 cents (Hong Kong 1977)

The song *Three Coins in a Fountain* was a top hit, sung by Frank Sinatra, in 1954; the lyric may still be found on YouTube. The three coins illustrated in Figure 4, representative of those that can be found in the Trevi Fountain, suggest a number of questions that continue to attract attention:

(i) does the roughness of a disc and the roughness of its rim (if serrated) influence its mode of descent in a tank of water?

(ii) what if the disc has a hole in it? It is to be expected that this may modify the wake quite dramatically, and so the various instabilities,

(iii) what if the disc has a wavy edge? Since the vortex shedding process occurs at the edge, this also may be expected to have a strong effect,

(iv) are there additional modes as yet undiscovered; it seems more than likely!

(v) with increasing Reynolds[7] number, does the wake become fully turbulent? If so, does the disc respond perceptibly to this turbulence?

These questions are very much in the spirit of Maxwell's original investigations with the slip of paper; as usual, his intuition was amazingly ahead of his time! To answer them will require further deep analytical, computational and experimental probing of the vortex shedding phenomenon.

3 Auguste, F. (2010), *Instabilité de sillage et trajectoires d'un corps solide cylindrique immergé dans un fluide visqueux.* PhD Thesis, Université Paul Sabatier, Toulouse, France - available at https://www.imft.fr/Projet-ANR-OBLIC

4 Auguste, F., Magnaudet, J. & Fabre, D. (2013) Falling styles of discs. *J.Fluid Mech.* 719, 388-405.

5 Y. Wang, C. Shu, C. J. Teo, L. M. Yang. (2016) Numerical study on the freely falling plate: effects of density ratio and thickness-to-length ratio. *Phys. Fluids*, 28 (10): 103603.

6 Field, S., Klaus, M., Moore, M. & Nori, F. (1997) Chaotic dynamics of falling discs. *Nature*, 388, 252–254.

7 A dimensionless constant representing the ratio of the inertial force (the force required to move the object) to the viscous force.

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Maxwell on Physical Standards

by **D. O. Forfar, MA, FFA, FRSE**, Chairman of the Clerk Maxwell Foundation

Maxwell's view on how standard units should be defined

In his 1870 Presidential address to Mathematical and Physical Sections of the British Association, Maxwell said:

"If, then, we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wavelength, the period of vibration and the absolute mass of these imperishable and unalterable and perfectly similar molecules."

In his celebrated treatise of 1873, *'A Treatise on Electricity and Magnetism'*, Maxwell, when talking about a standard for length, said (using a little of that pawky humour for which Maxwell was so famous):

"Such a standard would be independent of any changes in the dimensions of the earth and should be adopted by those who expect their writings to be more permanent than that body".

Within the length, mass, time (LMT) system of measurements there are many subsystems each of which has standard definitions for physically measurable quantities (e.g. the MKS-subsystem has standard definitions for a 'metre' of length, a 'kilogram' of mass and a 'second' of time).

In respect of a unit of length, the 'metre' was defined, in 1793, as one ten-millionth of the distance from the Equator to the North Pole. In 1799, it was redefined based on the distance between two marks on a bar held in Paris (the actual bar used being changed in 1889). In 1960, the metre was again redefined in terms the wavelength of a particular emission of light emitted by krypton. In 1983, the metre was again redefined as the distance travelled by light in vacuum in 1/299,792,458 of a second. As a result, the numerical value of the speed of light, namely 299,792,458 metres per second, is fixed exactly by this definition of the metre.

In respect of a unit of time, the 'second' was defined in terms of the *'mean solar day'* being a 1/86,400.002 of the *mean solar day* (a solar day is not quite 24 hrs as 24 (hrs)*60 (minutes)*60 (seconds)=86,400 which is not quite 86,400.002). A clock running at a constant rate (e.g. completing the same number of pendulum swings in each time period) cannot follow the actual Sun; instead it follows an imaginary *'mean Sun'* that moves along the celestial equator at a constant rate that matches the real Sun's average over the year (*'mean solar time'*). However, mean solar time is still not perfectly constant from one century to the next. The 'second' is now redefined in terms of the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

In respect of a unit of mass, in 1795, the 'kilogram' was defined as 1,000 grams, the gram being defined to be *'the absolute weight of a volume of pure water equal to the cube of the hundredth part of the metre, and at the temperature of 0°C (altered in 1799 to 4°C)'*. Also in 1799, an all-platinum kilogram object (called the *Kilogramme des Archives*) was fabricated with the objective that it would equal, as close as was scientifically feasible for the day, the mass of one cubic decimetre of water at 4°C. The kilogram was then redefined as being equal to the mass of the *Kilogramme des Archives*. This standard stood until 1889 i.e. for 90 years. In 1883, an object called the *International Prototype Kilogram* (IPK) was fabricated and its mass found to be indistinguishable from that of the *Kilogrammes des Archives*. In 1889, the mass of the IPK object was formally ratified as the 'kilogram'. The IPK is kept in Paris. However, in 2014, the General Conference on Weights and Measures (responsible internationally for the definitions of physical standards) accepted a resolution to *'take note of an intention'* that the kilogram be defined in terms of the Planck constant, *h*. Although it was recognised that significant progress had been made, they concluded that the data did not appear sufficiently robust to adopt the revised definition and that work should continue to enable the adoption in 2018.

It is expected that, by 2018, the definition of the metre, kilogram and second should all be based on the fundamental constants of nature. Standard units can therefore be reproduced in different laboratories by following a written specification. Maxwell's recommendation of 1870, on how standard units should be defined, will finally have been realised.

Dimensional Analysis

The first table of dimensional analysis was given by Joseph Fourier (1768 –1830) in his celebrated book *"The Analytical Theory of Heat"* (page 130). Fourier took the *fundamental-units* as a unit of temperature (θ), a unit of length (L) and unit of time (T).

Adding a unit of mass (M) makes up the fundamental-units. *Derived-units* are defined in terms of *fundamental-units* and therefore can be analysed dimensionally. For example, a unit of velocity is defined as the speed required to cover a unit of length in a unit of time (hence the dimension of velocity is $L*T^{-1}$). A unit of acceleration is defined as an increase in one unit of velocity in a one unit of time (hence the dimension of acceleration is $L*T^{-2}$). A unit of force is defined as the force which, when acting on a unit of mass, gives an acceleration of one unit of acceleration (hence the dimension of force is $M*L*T^{-2}$). A unit of work is defined as the amount of work which is done by a force of one unit acting over a unit of length (hence the dimension of work is $M*L^2*T^{-2}$).

Domestic-quality 1 kilogram cast iron weight with credit card to show the scale. Courtesy of Martinvl via Wikimedia Commons.

Maxwell on the dimensional analysis of electricity and magnetism

In 1863, Maxwell and his scientific colleague, Professor Jenkin (Professor of Engineering at Edinburgh University) wrote an article *"On the Elementary Relations between Electrical Measurements"* as an Appendix to the 2nd Report of the Committee on Electrical Standards[1].

The article covered the fact that, for electric and magnetic phenomena, there are two systems of measurement namely the electrostatic system (ESU) and the electromagnetic system (EMU). Derived units for the same physical entity (e.g. electric current), but defined on the two different systems, have different dimensions.

If the electric force is taken as fundamental (the ESU system), the force between two electric charges[2] gives rise to a definition of a unit of electric charge (defined so that two identical electric charges, each of unit strength, repel each other with a unit of force provided they are a unit of distance apart). As a unit of force has dimension $L*M*T^{-2}$, the dimension of an ESU unit of electric charge is thus $L^{3/2}*M^{1/2}*T^{-1}$. A unit of electric current can be defined by noting that a unit current is a unit quantity of electric charge flowing per unit of time and therefore has dimension $L^{3/2}*M^{1/2}*T^{-2}$.

As an alternative, the force of magnetism can be taken as fundamental (the EMU system). The force between two magnetic poles[3] gives rise to the definition of a unit of magnetic pole strength (two identical magnetic poles, each of unit pole strength, repel each other with a unit force provided they are a unit distance apart). Magnetic pole strength thus has dimension $L^{3/2}*M^{1/2}*T^{-1}$. A current (assuming flow round a wire bent into a circle) produces a force on a magnetic pole at the centre of the circle. A unit of electric current can be defined as that current that produces a unit force on a magnetic pole of one unit of magnetic pole strength situated at the centre of a circle (of unit radius) per one unit of length of the wire. The dimension of a unit of electric current is therefore $L^{1/2}*M^{1/2}*T^{-1}$ and a unit of electric charge (being a unit of electric current flowing for one unit of time) has dimension $L^{1/2}*M^{1/2}$. The unit of electromotive force is defined as that electromotive force which, in order to transfer one unit of electric charge between the terminals of a battery, takes one unit of work (hence electromotive force has dimension $L^{3/2}*M^{1/2}*T^{-2}$). The ratio of electromotive force to electric current (i.e. resistance) therefore has dimension $L*T^{-1}$, which is the same dimension as velocity.

Thus, a unit of the same derived physical object (in this case electric current) can be defined in two different ways and give rise to two different dimensional analyses, namely $L^{3/2}*M^{1/2}*T^{-2}$ on the ESU system and $L^{1/2}*M^{1/2}*T^{-1}$ on the EMU-system. The dimensional analysis for the ratio of the ESU to the EMU system also gives $L*T^{-1}$ (a velocity).

This ratio was first determined experimentally by Michael Faraday[4] and re-determined by Weber and Kohlrausch[5] in 1856 and 1857 as 310,740,000 metres per second (on the MKS-system) and further re-determined by Maxwell[6] in 1863/64 as 288,000,000 metres per second, almost equal to the then known velocity of light (and within 1% and 4% of the latest figure for the velocity of light). One unit of electric charge (on the EMU-system) is therefore exceptionally small (in relation to a unit electric charge on the ESU-system).

Maxwell and Jenkin give a table of the comparative dimensions of ESU and EMU units, of which the table below is an extract.

Physical object	ESU system	EMU system	Ratio ESU/EMU
Electric charge	$L^{3/2}*M^{1/2}*T^{-1}$	$L^{1/2}*M^{1/2}$	$L*T^{-1}$
Magnetic charge	$L^{1/2}*M^{1/2}$	$L^{3/2}*M^{1/2}*T^{-1}$	$L^{-1}*T$
Electric current	$L^{3/2}*M^{1/2}*T^{-2}$	$L^{1/2}*M^{1/2}*T^{-1}$	$L*T^{-1}$
Electromotive force (voltage)	$L^{1/2}*M^{1/2}*T^{-1}$	$L^{3/2}*M^{1/2}*T^{-2}$	$L^{-1}*T$

1 Maxwell J. F. and Jenkin F. (1863), *On the Elementary Relations of Electrical Measurements*, 2nd Report of the Committee for Electrical Standards, Appendix C.

2 Force= $q1*q2*r^{-2}$

3 Force= $m1*m2*r^{-2}$

4 Faraday, Experimental Researches, series iii, para. 361 et seq.

5 Weber and Kohlrausch (1857), Abhandlungen der König. Sächsischen Ges. Bd. ii, p.20 and Poggendorff's Annalen, (1856), Bd. xcix, p.110.

6 Maxwell J.C. (1863), *Experiments on the value of v, the ratio of the Electrostatic to the Electromagnetic unit of Electricity,* in 'Reports of the Committee on Electrical Standards' by Sir. W. Thomson, J. Clerk Maxwell et. al.

Isaac Newton's Apple Tree

Readers may be interested to know that the famous apple tree is still blooming at Woolsthorpe Manor (near Grantham, Lincolnshire) where Newton (1642–1727) was born and later inherited. The apple tree, although it has not been immune from the ravages of time, is still producing apples (Flower of Kent) after 350 years.

In 1665–66, Newton returned to Woolsthorpe Manor from Cambridge (on the outbreak of plague) and famously worked out his most world-changing theories, including his theory of universal gravity, the theory of light and the theory of calculus.

Newton's house, garden and small museum are now owned by the English National Trust.
https://www.nationaltrust.org.uk/woolsthorpe-manor.

Following a visit to Woolsthorpe Manor by the Chairman of the Foundation, the guardian at Woolsthorpe has very kindly offered the Maxwell Foundation (when the apples from the tree are ready for harvesting) some seeds from Newton's very apple tree. These will be planted in the garden of the house in which Maxwell was born (and later inherited) and which is the home of the Maxwell Foundation. Assuming the seeds 'take', then, in a few years, visitors to the Maxwell house should be able to see, growing in Maxwell's garden, a tree which has been pollinated from the seeds of Newton's apple tree.

Newton's Apple tree (courtesy, Ann Moynihan, Woolsthorpe)

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The birthplace in 1831 of James Clerk Maxwell.

www.clerkmaxwellfoundation.org

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