Reciprocal Figures for determining forces in framed structures

About 1864, the increasing use of framed structures, such as bridges and roof trusses, designers adopted graphical methods for determining the forces in members under given loadings. Maxwell developed this practice by establishing its theoretical basis in terms of *Reciprocal Figures* and promoted its use as a valuable design aid.

For the general case Maxwell stated "*Two plane figures are reciprocal when they consist of an equal number of lines, so that corresponding lines in the two figures are parallel, and corresponding lines which converge to a point in one figure form a closed polygon in the other. If forces represented in magnitude by two lines of a figure be made to act between the extremities of the corresponding lines of the reciprocal figure, then the points of the reciprocal figure will all be in equilibrium under the action of these forces.*"

Maxwell appreciated the work of Robert Bow and Fleeming Jenkin in popularising the reciprocal diagrams. Jenkin correctly thought Maxwell's abstract account was "*hardly of much help to a practical engineer*". However, Maxwell was of more practical help with his example of a roof truss design, shown on the right, published in The Engineer in 1867.

Allan Stewart (1831-94), a school friend and university contemporary of Maxwell, and later a leading designer of framed structures and Chief Assistant Engineer for The Forth Bridge, used this method in determining the forces for both the original Tay Bridge and Forth Bridge designs. In a report, on Sir Thomas Bouch's proposed Forth Bridge design, the eminent engineers William Barlow and William Pole found their investigations into "*strains … much facilitated by the use of the system of diagrams of forces first designed we believe by Maxwell, applied by Mr. Stewart to the present case with much skill.*"

The reciprocal figure determination of the forces in Bouch's Forth Bridge design is shown here. This project was abandoned in 1880 not from any shortcoming indicated by Stewart's assessment, but from a loss of confidence in Bouch and his work following the disastrous 1879 failure of his Tay Bridge.

Maxwell was awarded by the Royal Society of Edinburgh their 1870 Keith Medal for his work on reciprocal figures. These diagrams continued in use until the 1950's when increasingly they were replaced by other methods.

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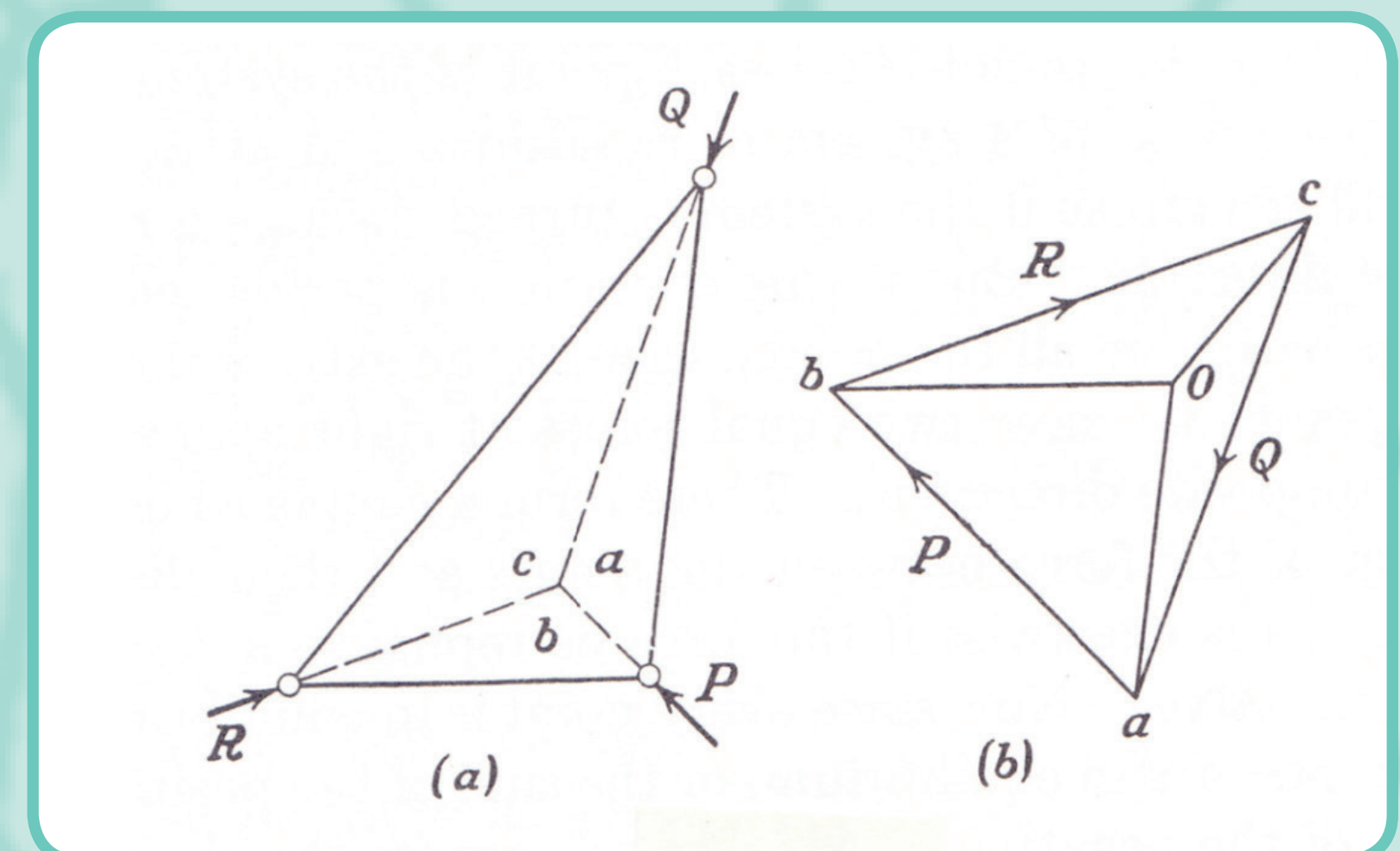
Simple example of reciprocal diagrams for a plane triangular frame in (a), after Maxwell. The three forces *P*, *Q* and *R* are in equilibrium and the forces in the bars in (b) are determined by constructing and scaling off their magnitudes [*0a*, *0b*, *0c*].
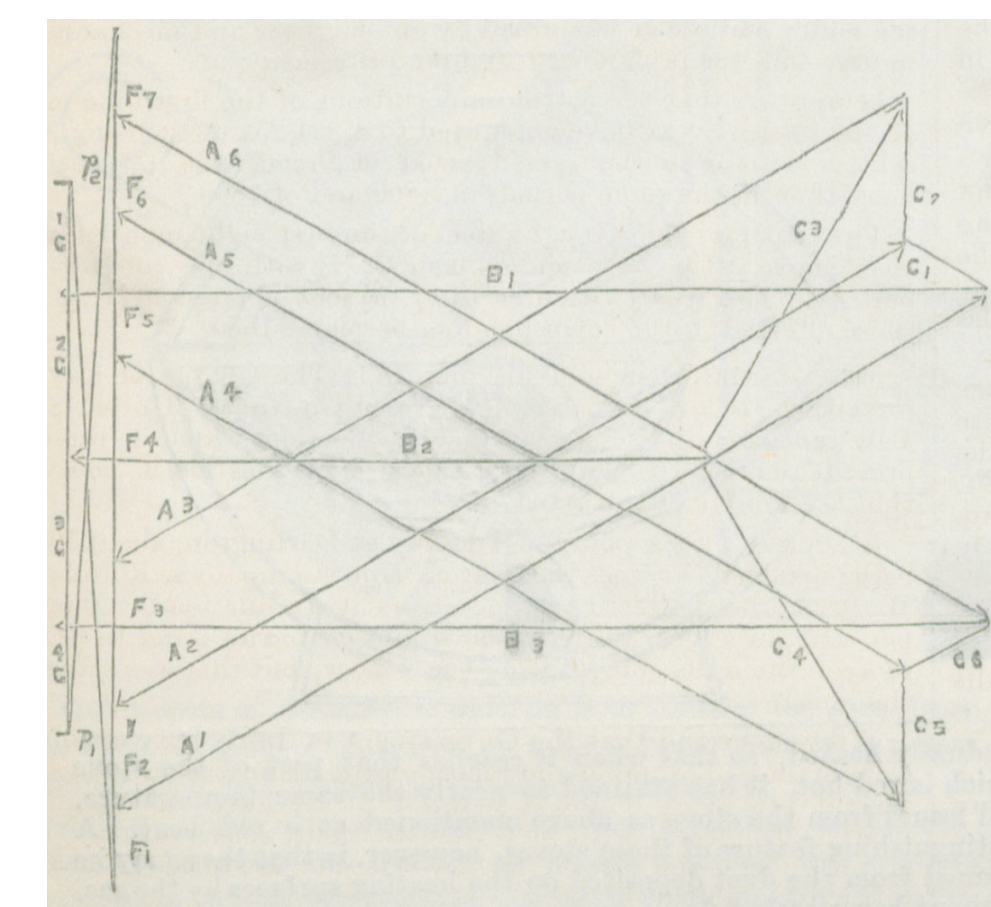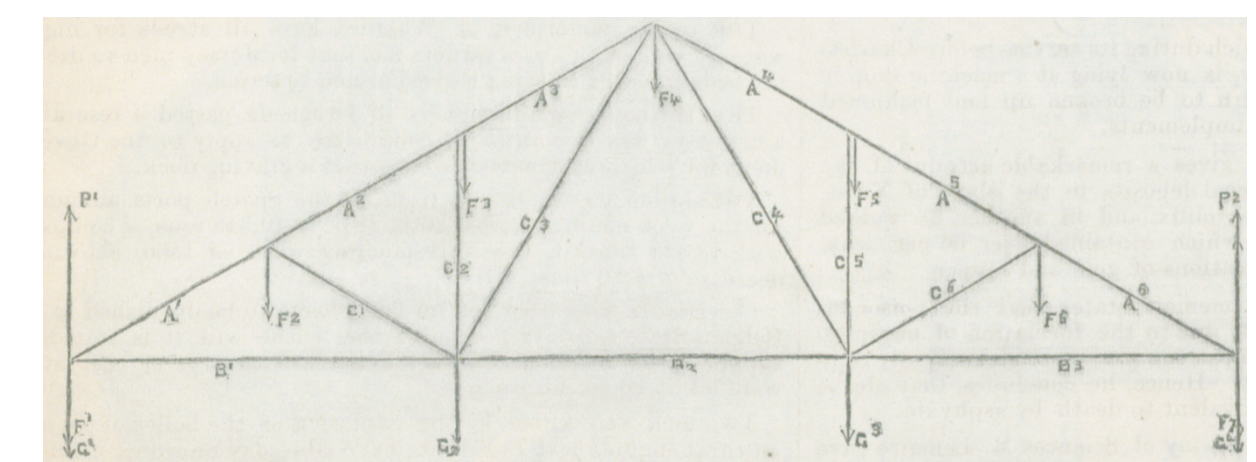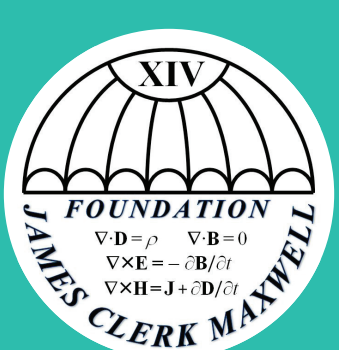

Maxwell's roof truss design in upper figure the forces in which were scaled from the lower force polygon.
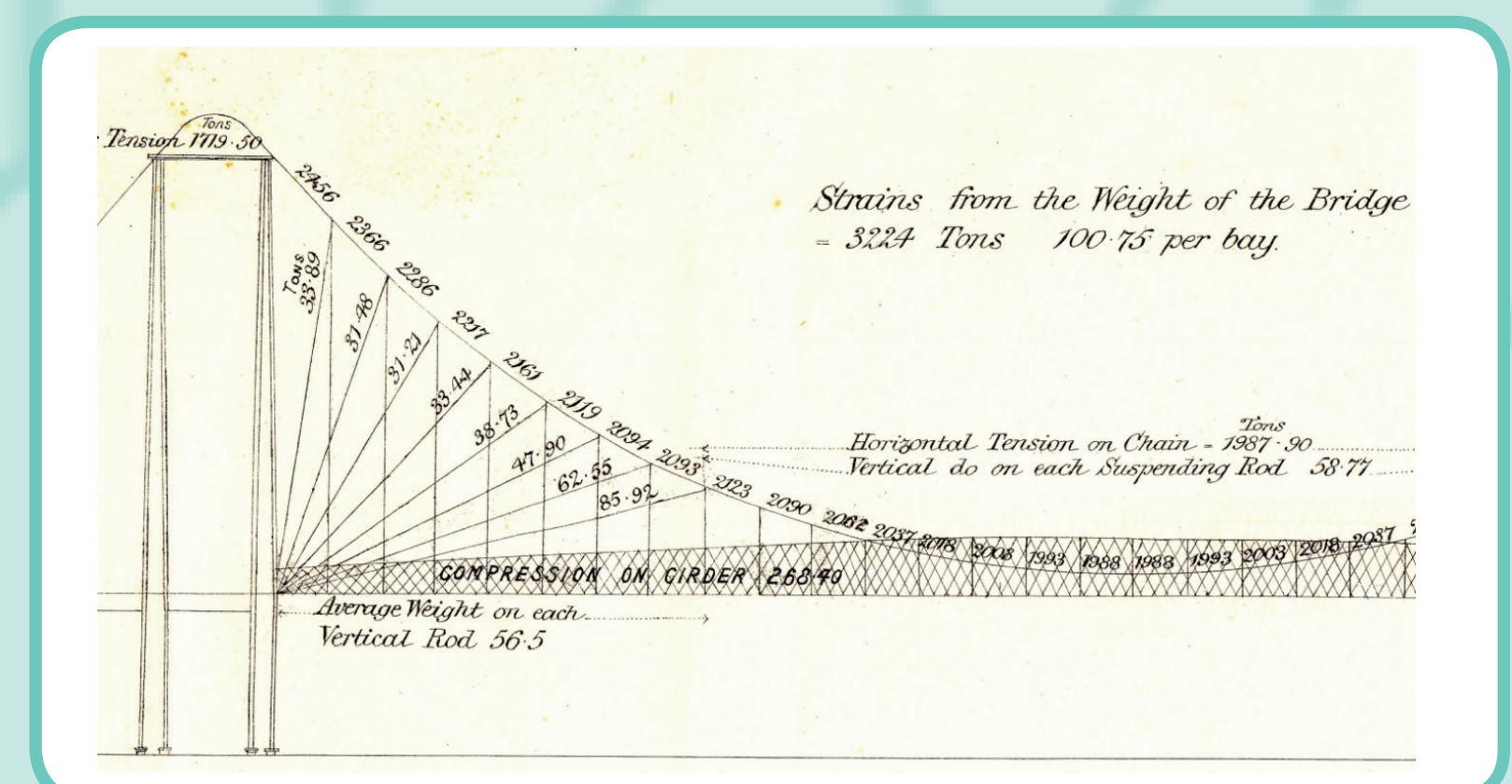
Stewart's reciprocal figure determination of the forces in Bouch's proposed Forth Bridge design.

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