



# newsletter

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## Maxwell and the Rings of Saturn

by Professor Andrew Whitaker, Emeritus Professor of Physics, Queen's University, Belfast



*A view of the rings of Saturn (from Gribbin, John and Goodwin, Simon (1998). Empire of the Sun: Planets and Moons of the Solar System. Constable, London.)*

Maxwell considered the rings of Saturn to be 'the most remarkable bodies in the heavens', second only, in his opinion, to the spiral nebulae. Yet it seems extremely unlikely that he would have devoted a substantial amount of time and effort to determining their structure mathematically had the topic not been chosen in 1855 as the subject of the Adams Prize of 1856.

The deadline for submission was December 1856, and Maxwell submitted his attempt – the only one – just before the deadline. He was awarded the prize in June 1857, but continued to work on the topic for a further year, finally publishing his results as a book in 1859.

This was at a time when Maxwell had a great range of other interests and duties. From March 1855, he must certainly have been absorbed by his first studies on electromagnetism. In April 1856 his father died, and even apart from the time required for grieving, he spent much effort taking over the charge of the family estate. Also, in February 1856, just before his father died, he had applied for the vacant Chair of Natural Philosophy at Aberdeen's Marischal University, requiring him to take time requesting testimonials. He was duly appointed and took up his post in November of that year, and this, of course, required him to prepare teaching and also to write and present an inaugural lecture.

Against this background, why was Maxwell prepared to put in so much work over several years on the rings of Saturn? Admittedly the prize was substantial – £130, or perhaps £13,000 in today's terms, and this was rather more than a third of his full year's salary at Aberdeen. But it is more likely that the reasons for taking on the challenge were, firstly, personal prestige, but, secondly, and even more importantly, the wish to maintain or increase the scientific stature of Britain and in particular Cambridge University.

The Adams Prize was instituted in recognition for the contribution of the young Cambridge mathematician John Couch Adams, made only eight years earlier, to the discovery of the new planet Neptune. His contribution, though, was disputed, particularly in France. Adams had told the senior Cambridge observational astronomers, George Biddell Airy and James Challis, of his prediction of the position of the planet as early as September 1844, but he had not published his work, and no search had been made for the planet before the French mathematician Urbain Le Verrier published a similar prediction, and was also able to persuade the German astronomer Johann Galle to search for the planet. Galle speedily met with complete success in September 1847. To put it mildly, French scientists were not convinced that Adams deserved to share the honour of the discovery with

Le Verrier, or to be worthy of the credit bestowed on him by Britain and, in particular, Cambridge.

The funds for the Adams Prize were provided by alumni of Adams' Cambridge college, St. John's, and the prize was in the gift of the University. Airy and Challis were heavily involved in choosing topics in the first few years of the prize, and in Maxwell's year William Thomson was also an examiner. Thomson, later Lord Kelvin, had, of course, been Professor of Natural Philosophy at Glasgow University since 1846, but retained very strong links with Cambridge throughout his career. Thus the institution of the Adams Prize and the quality of the prize-winning essays may be regarded as an element of academic rivalry with France, and it is quite likely that Maxwell saw his heavy labours in this way.

As early as 1610, Galileo had observed structure about Saturn, though it was Christian Huygens in 1656 who was able to identify its ring nature and Jean Cassini, a century later, who discovered that there were, in fact, two concentric rings separated by a dark band. Shortly before the setting of the Adams Prize topic, several astronomers identified an inner dark ring, which was partially transparent.

There were, of course, many ideas about the structure of the rings, in particular whether each ring was solid or consisted of a large number of rocks. However there was very little rigorous mathematical analysis, and, as Airy pointed out, even the conclusions of the great mathematician Pierre-Simon Laplace were somewhat flawed, though many of his arguments were valid – he showed that the rings could not be a uniform solid but claimed they therefore must be an irregular solid.



So we may consider Maxwell's work as detailed rigorous and comprehensive mathematics on an essentially blank canvas.

Maxwell's methods were not actually very advanced, consisting of potential theory, mainly using linear differential equations, Taylor series and Fourier analysis, but the analysis was sophisticated and exceptionally sure-footed. In general, he searched for equilibrium configurations of planet and ring, and then used Taylor series to study whether any such configuration might be stable to small disturbances.

Maxwell was first able to show that the cases of a uniform ring or a non-uniform ring did not lead to stable solutions. He did, though, find a highly bizarre case of stability – a uniform ring loaded at a particular point with a very heavy weight. The conditions for stability of this, somewhat odd, type of ring were stringent; the ratio of the displacement of the centre of gravity of the ring to its radius must be between 0.8159 and 0.8279, and the weight at the special point must be between 4.43 and 4.81 times that of the rest of the ring. Amusing as this solution was, it was clear from direct observation that it bore no relation to the actual ring system, so Maxwell's overall conclusion was that each ring was not a solid system.

Maxwell now considered the possibility that the ring system was fluid, which initially attracted him. Naturally in order to balance centrifugal effects, the ring must rotate. His method was to analyze wave motion in the ring and to consider whether such waves will be stable. Let us imagine, for example, a place where, as part of the wave motion around the ring, the ring is thick, and for stability, fluid should move away to the

neighbouring regions, the system moving towards equilibrium. Yet Maxwell's mathematics shows the opposite effect occurring.

Suppose a portion of fluid needs to increase its speed around the planet in order that the system should move towards equilibrium. As it increases its speed, the balance between gravitational attraction and centrifugal force is broken, and this portion of fluid moves outwards. Thus its path has a larger circumference and so its angular speed decreases. So compared to adjacent portions of the ring, it is going backwards, and the expected catching up is totally stymied. Thus the disturbance does not result in a restoring effect but in an increasing divergence from equilibrium. In other words a fluid ring system is intrinsically unstable. Maxwell was to write that: "*This effect is very remarkable as showing the destructive effect of an apparently conservative force.*"

As Maxwell pointed out, the fact that Saturn itself could be seen through the inner dark ring in its true position indicated the correctness of this suggestion. The light must be passing through gaps in the ring; if the ring were fluid the image would be refracted. Thus the only possibility seems to be that each ring consists of a very large number of unconnected satellites. Now the challenge was for Maxwell to show the stability of such a system.

Maxwell modelled the situation with a system of equivalent rocks or satellites spaced equally round the ring, and calculated whether any disturbance would lead to a wave travelling round the ring or a total disruption of the system. At first sight, the second possibility seemed likely, for if a particular satellite is disturbed so as to be

nearer one of its neighbours, it would seem that gravitation would act to decrease further the distance between the two satellites. But the reverse occurs, and for the same reason discussed in the fluid case. When a satellite speeds up, it swings out from its orbit, and so essentially its angular speed decreases. The situation is indeed stable.

It must be remembered that there is more than one ring, and Maxwell proceeded to investigate a system of two rings, each of the same form as the single ring just discussed. Complicated phenomena may occur; if, for example, a wave in one ring is in resonance with a different type of wave in the other ring, a combined wave may increase without limit leading to collapse of the ring system. Maxwell argued, though, that the ring system could exist for a very long time without such destruction.

Maxwell knew, of course, that his regular rings were an over-simplification, and attempted to generalize to an irregular distribution of satellites; it seems likely that these thoughts led to his important work on kinetic theory.

Maxwell's explanation has broadly been found to be correct by the Voyager mission of the 1980s and the Cassini probe of 2009. Each ring consists of an immense number of particles – mostly ice but some rocks – each rotating at the appropriate speed for its distance from Saturn. Each ring, though, consists of much variation of brightness – maxima and minima with total gaps and ringlets of very high density. It is nice to note that there is a *Maxwell gap* and a *Maxwell ringlet*. ■

## William Dyce Cay (1838-1925), Civil Engineer *James Clerk Maxwell's mathematical cousin*

by Professor Roland Paxton, MBE, FICE, FRSE, Hon. Professor Heriot-Watt University; Vice-Chair, Institution of Civil Engineers (ICE) Panel For Historical Engineering Works ; Trustee of the James Clerk Maxwell Foundation



William Dyce Cay © ICE Archives

William Dyce Cay, second son of Robert Dundas Cay, W.S., lawyer and brother of Frances Cay the mother of James Clerk Maxwell (1831 – 79) was born on 28 March 1838 at 18 Rutland St. Edinburgh. He was educated at the Rev. Wm. Bliss's school, Peebles from 1844–53 and Edinburgh University where, in 1856, he obtained its highest mathematical prize, the Straiton Gold Medal and 1st prize in the 2nd Division of the Natural Philosophy class.

In obtaining these awards, Cay acknowledged that the '*tuition and example*' gained from his cousin (Maxwell) on long walks from the estate at Glenlair, Kirkcudbrightshire, in 1855 had '*had good effect*'. This was even though '*before I had got to the bottom of one example he had rushed off into another!*' In 1856 Maxwell went with Cay to Belfast and introduced him to his friend James Thomson (elder brother of William, later Lord Kelvin), Engineer to Belfast Waterworks and, from 1857, Professor of Engineering at Queens College, Belfast.

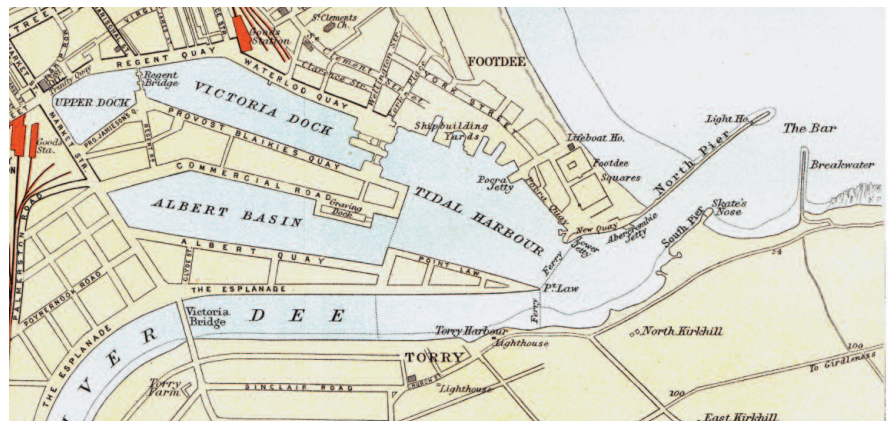


Cay's Glenlair road bridge – 45ft span © K. McCrae

Cay served as a pupil under Thomson from 1856–58, completing his four-year apprenticeship in November 1860 as an assistant resident engineer with leading Edinburgh consulting engineers B. and E. Blyth. He worked on the construction of the 60-mile Portpatrick Railway (from Castle Douglas) from the firm's Creetown site office and afterwards in its Edinburgh office. Interestingly, a resident engineer contemporary on this project was Maxwell's fellow Edinburgh Academical, Allan D. Stewart (1831–94 – a Cambridge University 9th wrangler in 1853, the year before Maxwell became 2nd wrangler). Stewart later contributed significantly to the design of the Tay and Forth Bridges, including the Forth suspension bridge (abandoned in the wake of the Tay Bridge failure) in the feasibility report on which it was noted that he applied 'Professor Clerk Maxwell's diagrams of forces ... with much skill'.

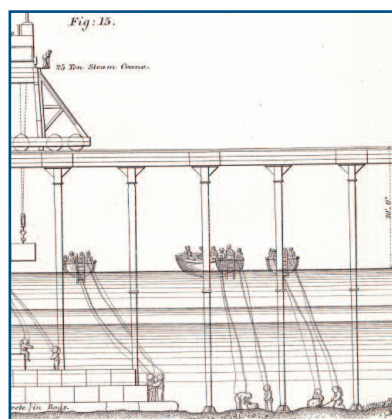
From June 1862 to August 1863 Cay worked as resident engineer on the Turin and Savona Railway under its chief engineer James Abernethy, then for two years in London as an assistant to A.M. Rendel, returning to Scotland from 1865–67 as chief assistant to James Leslie, Edinburgh Waterworks Engineer. In a testimonial to Leslie, Maxwell, who had returned to Glenlair after resigning his Natural Philosophy chair at King's College, London, wrote of Cay:

*'His mathematical knowledge is sound and he has made a special study of iron and stone bridges, some of the results of which he has been in the habit of communicating to me ... A skew bridge had to be erected, but some of the stones ... were not properly shaped. Mr. Cay however designed the correct form ... and the bridge was erected according to his plans. Mr. Cay has made a design of a bridge for me across the the River Urr which I intend to have executed next Spring' (1866 – still in service, see above).*

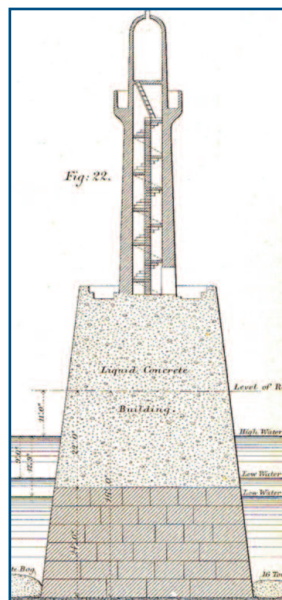


Aberdeen Harbour after improvement Groome's Gazetteer 1882

Cay then embarked on his most important civil engineering work as Resident Harbour Engineer at Aberdeen from 1867–80. His direction of major improvements to the harbour (see map) included channelizing about a mile of the river Dee and construction of the present South Breakwater (1050ft long) – see illustrations.



South Breakwater being built under Cay's direction MPICE 39



Breakwater end and Lighthouse

In constructing the harbour entrance, still in service, Cay adopted concrete, in preference to traditional rubble and masonry, using about 15,000 tons of Portland Cement in liquid concrete and large blocks. Full width single pours ranged up to a then remarkable 1,300 tons. He designed the plant and staging and implemented, from 1872, a new mode of depositing liquid concrete from a hopper barge to 20ft below Low Water in jute bags holding up to 100 tons.

Cay also designed steam-operated barges for carrying dredged material and a concrete mixing machine. His experimental work included quantifying the increased early strength of cement briquettes made with salt water. Although he did not invent the concrete bag technique, he was the first to use it on a large scale to fit a breakwater sole neatly to an undressed firm foundation, minimising costly excavation. Cay's achievement at Aberdeen was illustrated in Vernon-Harcourt's Harbours and Docks classic text book and, that he deserved 'great credit', was also publicly acknowledged by other eminent engineers including Abernethy and Sir John Hawkshaw.

Cay's innovations earned him a bronze medal at the Royal Mining Engineering and Industrial Exhibition at Newcastle-upon-Tyne in 1887 and a Brisbane Medal from the Royal Scottish Society of Arts in 1888. By then his concrete in bags technique (although of limited later application as sheet steel and the use of compressed air developed) had been used at Buckie, Lerwick, Arbroath, Fraserburgh and Newhaven (Sussex) harbours, and New Plymouth Breakwater (Ngamotu), New Zealand.

By the 1870s Cay had become a leading member of his profession.



He was elected to the Institution of Civil Engineers (AM, 1864; M, 1872); FRSSA (1867) and FRSE (1882). From 1880–1907 he practised in Edinburgh as a consulting engineer, firstly at 8 South Charlotte Street, from 1882–98 at 107A Princes Street and then from 1 Albyn Place. Work on which he was engaged included harbours at Buckie and Lerwick, reconstruction of the dock entrance at Arbroath and advising on Royal Bridge over the Dee at Ballater. In 1908 he went to London and, for about ten years, took an office at 39 Victoria Street.

Cay's progressive practice formed the subject of his influential papers published by the Institution of Civil Engineers (ICE) and Royal Scottish Society of Arts. His career was also remarkable for the large number of valuable contributions he made to ICE Proceedings from 1873–1922. Subjects included stability of channels and walls, sand travel, training of rivers, bars at mouths of tidal estuaries, reduction of wave action, coast erosion, use and testing of concrete, groynes, harbour layout and dredging. Many related to specific projects at home, also in Holland, France, Panama, Japan, South Africa, Ceylon, Burma and India including a £760,000 proposal for an outer harbour at Madras. He also wrote technical articles for Chambers' Encyclopaedia.

In his 'Recollections' of Maxwell, Cay notes that he (Maxwell) was 'very fond of society and conversation ... He took much pleasure in my studying and was unwearied in explaining difficulties and wrote out for me a treatise on hydrostatics, hydrodynamics, statics, dynamics and solid geometry ... Although clear on paper he was not so in viva-voce explanations'. '1857, I went to be best man at his marriage at Aberdeen'. 'When he was professor at King's College, London, I remember his explaining to me the reasons which made him give up that post viz. that the students did not care for instruction except in engineering or practical mathematical subjects'.



Cay's Breakwater still protects Aberdeen Harbour after 140 years Postcard 1910

Cay never married and lived during the latter part of his life at the Junior Carlton Club in London or at hotels in various places. He was a member of this and numerous other clubs including the Oriental Club, London; the University Club, Edinburgh; the Royal Northern Club, Aberdeen; the Hon'ble Company of Edinburgh Golfers, Musselburgh; the Aberdeen Golf Club; and of the Casino, Savona. He died on the 2 December 1925 in a nursing home at Folkestone after a short illness. ■

**Main sources:** ICE Archives; MS. Abstract from Cay's 'Family Book' (1865), Testimonial by Maxwell for W.D. Cay, 26 Sept. 1865, 'Recollections of James Clerk Maxwell (1854-64) by W.D. Cay, 30 April 1881', for copies of which the author thanks James Brown (Cay relative); *The Times* 4 Dec. 1925, 1; *Engineering* 11 Dec. 1925, 749; PRSE XLVI (1925–26); TRSSA XII (1891); MPICE 37–212 (1873–1922); Vernon-Harcourt L.V. *Harbours ...*, 1885; Paxton R. & Shipway J. *Civil Engineering Heritage Scotland*, 2007.

## Book Review: *Faraday, Maxwell and the Electromagnetic Field*

by Forbes N. & Mahon. B. (2014), Prometheus books, Amherst, New York.

This excellent book weaves together the history of Faraday and Maxwell and the gradual uncovering of the secrets of magnetism and electricity. Today, we take for granted radio, television and mobile phones but it is too easily forgotten that, in the past, the secrets of magnetism and electricity were not known and, as the book shows, it required the immense creative efforts of many scientific men to uncover the secrets (e.g. Gilbert, Coulomb, Volta, Davy, Oersted, Ampère, Faraday, Kelvin (Thomson), Maxwell, Hertz, Heaviside, Fitzgerald, Lodge, Marconi). The story of the discovery of the secrets of electricity and magnetism, right up to the transmission of radio waves across the Atlantic by Marconi, makes for an exciting read.

The dedication of Faraday to the task of understanding the laws of Nature both by conducting his own experiments and by repeating the original experiments of others (which Faraday deemed essential), comes out very clearly in the book. The assiduousness of Faraday in carefully describing and noting down the results of his experiments (in his *Experimental Researches in Electricity*) is well highlighted. Maxwell, on the advice of Lord Kelvin, started his research by reading Faraday's *Experimental Researches in Electricity* and titled his first paper on electricity and magnetism 'On Faraday's Lines of Force'. The book reminds us that the general body of scientific opinion at the time rather dismissed Faraday's notion of 'lines of force' until Maxwell showed that it captured perfectly the notion of a 'field' and lent itself to a proper mathematical formulation.

Although Faraday was ignorant of mathematics, Maxwell said in his tribute to Faraday "The way in which Faraday made uses of his idea of lines of force ... shows him, in reality, to have been a mathematician of a very high order – one from whom the mathematicians of the future may derive valuable and fertile methods". That Maxwell himself, by way of 'Maxwell's Equations', gave mathematical expression to Faraday's concept of 'lines of force' and to the concept of 'fields' (both electric and magnetic) comes out very well in the book; as does Maxwell's addition (to the Oersted / Ampère Law) of a displacement current giving rise to a magnetic field whenever the electric field changes – symmetric to the way Faraday's Law shows that an electric field arises whenever a magnetic field changes. The book shows how, in regard to electricity and magnetism, Faraday stood on the shoulders of Oersted, Maxwell stood on the shoulders of Faraday and Einstein, in formulating his theory of 'Special Relativity', stood on the shoulders of Maxwell.

The history of Maxwell from his early up-bringing in Glenlair (in Galloway, Scotland) through to periods in Edinburgh, Cambridge, Aberdeen and London is well described in the book.

The model (of electricity / magnetism) contained in Maxwell's paper 'On physical Lines of Force' is well depicted, as is Maxwell's dispensing of this model in his subsequent crowning paper 'A Dynamical Theory of the Electromagnetic Field' which contains the famous 'Maxwell's Equations'

for the first time and with the consequence encapsulated in Maxwell's immortal words: "This velocity is so nearly that of light that it seems we have strong reason to conclude that light itself (including radiant heat and other radiations if any) is an electromagnetic disturbance in the form of waves...".

Furthermore, the book states that, in "his 'Dynamical Theory' paper, Maxwell heralded one of the very rare events (a paradigm shift) in science." i.e. Maxwell understood before most that Nature cannot properly be comprehended by only a mechanical explanation and that there is a world lying beneath the surface of everyday reality which requires a mathematical description.

The introduction to the book describes Hertz in his laboratory generating and detecting the very electromagnetic waves, travelling at finite speed, that Faraday had predicted in his 'Ray Vibrations' talk and that Maxwell had shown were a direct consequence of his equations. I think the reader will acknowledge that this was a triumphant conclusion of very fruitful and creative scientific endeavour.

Anyone who wants to read about Faraday and Maxwell and their contributions to electricity and magnetism culminating in Hertz's generation and detection of electromagnetic waves will enjoy this book and learn much from it.

**D. O. Forfar,**  
Chairman of the Clerk Maxwell Foundation