Aspects of the Life and Work of Peter Guthrie Tait, FRSE[1]

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Peter Guthrie Tait was born in Dalkeith in 1831. Upon the death of his father when he was just six his mother took him and his two sisters to live in Edinburgh with her brother. Here in his uncle's house he was encouraged to dabble in photography and astronomy. We know that at the age of thirteen he was making nightly observations of the positions of Jupiter's satellites. By this time he had entered the Edinburgh Academy. Fleeming Jenkin[2] was a member of the same class, James Clerk Maxwell in the class above. The friendship which developed between Maxwell and Tait during their school days would last throughout Maxwell's relatively short life. If there was a competitive element to that friendship, encouraged by the school's awarding of the mathematics prize to Tait in 1846 and to Maxwell the following year then it was certainly not apparent. They exchanged drafts of papers they were writing in their teens and we know that Tait retained his annotated copies of Maxwell's early geometrical papers for many years.

At sixteen both young men went up to their local university but after just one session Tait moved on to Peterhouse, Cambridge, from where he graduated as Senior Wrangler in 1852. When Maxwell eventually moved to Cambridge it was in Tait's college that he initially enrolled despite being advised by Forbes[3] to enter Trinity.

Within a couple of years of graduation Tait had been offered a mathematics chair at Queen's College, Belfast. His research interests, however, were still wide-ranging and he was in no way constrained by the teaching requirements. His first experimental work was undertaken in Belfast where he assisted Thomas Andrews in the preparation of three papers on ozone. He also re-established his friendship with the Porter brothers whom he had met at Peterhouse and, in 1857, he married their sister. Tait returned to Edinburgh to succeed Forbes as Professor of Natural Philosophy in 1860. His expertise as a lecturer gave him the edge over Maxwell, who also applied for the post.

According to Tait's contemporaries the new professor made an impression on all who met him. He had an immense presence. One of his protégés, Macfarlane, who was himself only just short of six-foot, felt dwarfed by the man. In the lecture theatre, whether addressing 200 undergraduates or a smaller number of advanced students, he would write out mathematical equations for an hour without referring to his notes and would deliver his sentences with clarity and purpose.

Upon taking up his post in Edinburgh, Tait discovered that he was required to engage in experimental rather than theoretical physics. However, in contrast to Glasgow where William

Thomson was professor, there was no physical laboratory at Edinburgh. Students lacked the facilities, not only to replicate the experiments demonstrated by Tait in his lectures but also to undertake original research. With the support of Sir David Brewster (1781-1868) the laboratory was up and running by the end of the 1860s but it was with the expansion onto the floor above a decade later, made possible by the relocation of the anatomy department, that the project was completed. Tait was at home in Edinburgh; he held a prestigious chair in a university which would yet see the likes of Max Born and Sir Edward Appleton and here he would stay for forty years.

Tait was a prolific writer both of scientific papers and of textbooks. Of the latter he wrote sixteen:-

A Treatise on Dynamics of a Particle (with W. J. Steele), 1856.
Sketch of Elementary Dynamics (with William Thomson), 1863.
A Treatise on Natural Philosophy (with William Thomson), 1867.
An Elementary Treatise on Quaternions, 1873.
Elementary Dynamics (with William Thomson), 1867.
Elements of Natural Philosophy (with William Thomson), 1873.
Sketch of Thermodynamics, 1868.
Introduction to Quaternions (with Philip Kelland), 1873.
The Unseen Universe (with Balfour Stewart), 1875.
Recent Advances in Physical Science, 1876.
Paradoxical Philosophy (with Balfour Stewart), 1878.
Heat, 1884
Light, 1884
Properties of Matter, 1885
Dynamics, 1895
Newton's Laws of Motion, 1899

Of these texts, one in particular deserves special mention. The *Treatise on Natural Philosophy* was the fruit of a collaboration between Tait and William Thomson (1824-1907), later Lord Kelvin. The two physicists began writing the book soon after Tait returned to his native land from Belfast, where James Thomson (1822-1892), Kelvin's brother, had been amongst his colleagues. The *Treatise on Natural Philosophy* appeared in 1867 as the first volume of what was intended to be an even larger work. Its reception could not have been better had it been written by Laplace or Lagrange, and its translation into German was quickly arranged by Helmholtz. Affectionately dubbed Thomson and Tait or even T and T' it is one of the classic science texts of all time.

Tait wrote some 133 papers and a further 232 popular articles, laboratory notes, reviews and tributes to other scientists alive and dead. There are three major papers on knots. No less than thirteen papers are devoted to the paths of spherical projectiles, especially golf balls, and one is on the application of probability to matchplay golf. There are about 70 papers on quaternions though just 25 of them specifically mention quaternions in the title. Each of these areas will be explored briefly, the last in the context of the controversies which Tait tended to court.

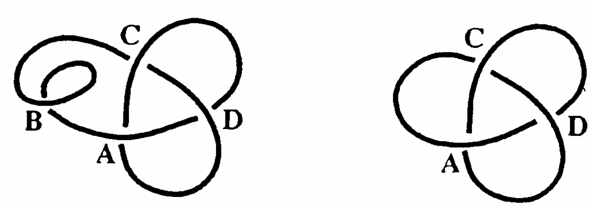
Topology of Knots

Tait's topology of knots, published in three papers between 1876 and 1885, was the second serious mathematical study of the subject. It sprang from the speculative vortex theory of the atom, propounded by Kelvin. Tait himself acted as a catalyst for Kelvin's theory in two ways: firstly by translating into English a paper of Helmholtz in which the apparent attraction and repulsion of vortices in certain fluids was noted and then by spotting the same phenomenon in the aerodynamics of smoke rings. And secondly, in the early weeks of 1867 he devised an apparatus for making smoke rings, an apparatus consisting of a large box, initially open at one end and with a circular hole in the opposite face. Kelvin was present when Tait demonstrated its use to his students. The box was filled with smoke and then a rubber sheet was stretched over the open end. By striking the rubber sheet smoke rings were produced at the hole. The way in which the rings intermingled and deflected impressed Kelvin so much that he decided to employ two boxes in his experiments in order to increase the number of collisions. Kelvin enunciated his vortex theory at a meeting of the Royal Society of Edinburgh on 18 February 1867.

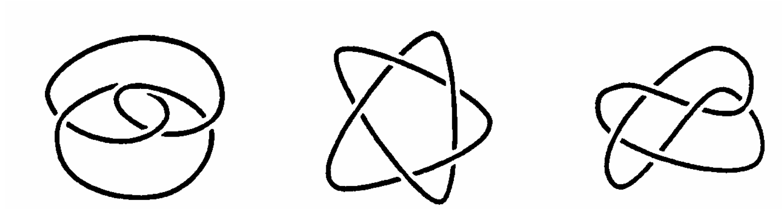
Now it seemed to Kelvin that there might be a one-to-one relationship between knotted smoke rings and atoms. If a full classification of knots could be worked out then the properties of all known matter might be discovered. He persuaded Tait and Crum Brown to trawl for all the knots up to a given complexity with a view to providing a full classification. Maxwell was enthusiastic. He wrote: *'May you......prosper and disentangle your formulae in proportion as you entangle your worbles'*.

The relationship with chemistry quickly disappeared from sight, but Tait's fascination with knots only grew. At the outset, Tait believed that no work had been done previously in this area, and it was not until he had made significant progress that Maxwell sent him a copy of the book *Introduction to Topology* written by a former student of Gauss, Johann Benedict Listing (1808-1882) in 1847.[4] Topology, as a mathematical term, was born in the title of this book! Tait acknowledged that although Listing had made no attempt to trawl for all possible forms, he had certainly anticipated his own investigations. Tait was clearly unaware at this time of Listing's second book of 1862, *The Census of Spatial Complexes*.[5]

Now, when a knot can be transformed into another by twisting and manipulation but without cutting then they are essentially one and the same. The first problem Tait encountered was how to tell whether two knots which appeared to be different were actually so. In his first paper he introduced the concept of knottiness, defining it as the minimum number of intersections a knot possesses. The trefoil has knottiness 3 for example, and therefore any knot which can be manipulated to form a trefoil also has knottiness 3.



Once transformed into their simplest diagrammatic forms each knot could be described in symbolic form dubbed a scheme. One method of moving from knot to scheme is to traverse the knot calling the first intersection met A, the second B (unless it is A again) and so on. For the first knot the scheme is ABBCDACD. For the second, simply ACDACD. Notice that the double B may be eliminated, as B is a 'nugatory' crossing, one that can be removed merely by twisting. This is the most obvious of a number of algebraic procedures which Tait developed for the simplification of a knot's projection. But he also added one from topology - the *flype*. *Flyping* is a Scots word associated with turning something inside out and, no doubt, Tait imagined the knot stretched around the surface of a glass sphere, so that it can be viewed from inside or outside. Armed with a variety of approaches to his investigation Tait went about finding all the different forms of knots of given knottiness.



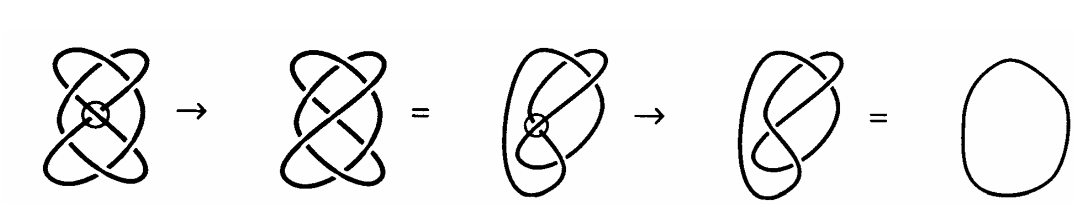
knottiness 4 ***knottiness 5***

knottiness 6

There are no knots with knottiness 1 or 2. The trefoil is the only knot of knottiness 3, while for knottiness 4, 5 and 6 there are one, two and four forms respectively. Tait also managed to find the eight forms of knot with knottiness 7.

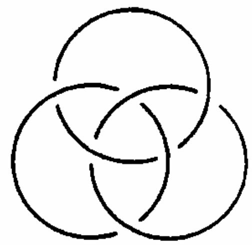
And there for a while he rested his case. However, after completing the paper Tait became aware of the investigations of Thomas Penyngton Kirkman (1806-1895) on the properties of polyhedra, research which seemed to parallel his own work. Kirkman had been able to confirm Tait's results up to sevenfold knottiness and had extended the census to knottiness eight and nine. Armed with yet another method for discovering new designs Tait was able to make one or two corrections to Kirkman's results and illustrate all the forms with knottiness eight or nine in his second paper, and with knottiness ten in his final paper of 1885.

In the months following the publication of the first paper Tait concentrated on a different concept, that of beknottedness or belinkedness. He defined it as the minimum number of cuts and reconnections required to reduce the diagram of a knot to a simple loop. In practice all possible configurations of the knot have to be checked to calculate this invariant. Below is a knot of beknottedness 2. Crossings which are to be altered are ringed and the equals signs indicate that one form may be manipulated into the other without cutting.



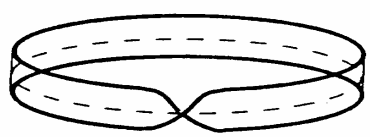
The task was not without its frustrations as we can see from a letter written by Tait to Maxwell in June 1877:

I have got so thoroughly on one groove that I fear I may be missing or unduly exalting something which will appear excessively simple to anyone but myself. You are just the party to detect this. Here for instance, is one of my difficulties. What are we to call the 'belinkedness' of the arrangement where there is no linking at all & yet you can't separate the rings? If you change any one sign a ring comes off, but one degree of linkedness is introduced! This is neither Knot nor Link. What is it?



Reading through Tait's papers for the first time it is surprising to discover that he gave a rather familiar treatment of the one-sided surface, the Möbius band. We can produce a Möbius band by taking a strip of paper, giving it a half-twist before connecting the ends. The discovery of this surface is one of the classic cases of simultaneous independent discovery. August Ferdinand Möbius found it around September 1858 when analysing the geometry of polyhedra in pursuit of a prize offered by the Paris Academy of Sciences but he did not publish until 1865. Listing found the same surface in July 1858 and brought it to public notice in 1861.

Why is Tait's treatment so familiar? Well it is precisely the treatment to be found in a host of books today: for example in the books on recreational mathematics written by Martin Gardner from his column in *Scientific American*. First cut the band along its length half way in. Then try a cut one-third of the way in and so on. Sometimes another band is created, perhaps with a different number of half-twists; sometimes there are two interlocking bands.



In his papers Tait credits neither Möbius nor Listing with the priority of discovering what happens when the band is cut, despite fully acknowledging Listing's work on knots. Setting the record straight, in a tribute to Listing in *Nature* in 1883, he noted that some of the band's idiosyncrasies had been in common currency among jugglers for some time and that Listing had alluded to them in his *Introduction*.

The mathematical study of knots mushroomed in the 1920s with the discovery of invariants called the Alexander polynomials after their discoverer, the American topologist, James Waddell Alexander. A second burst of activity in the 1980s was founded on the work of the New Zealander, Vaughan Jones, and led to breakthroughs by research groups around the world. In recent times the topology of knots has been applied to string theory, fluid flows and to DNA.

Golf Ball Aerodynamics

Throughout his four decades as Professor of Natural Philosophy at Edinburgh, Tait's sorties beyond the city boundaries were few and far between. When he did leave home it was for the links at St Andrews, where he played as many as five rounds in a day. This passion for golf rubbed off on his son, who became the leading Scottish amateur golfer of the last decade of the century. Freddie Tait reached the final of the British Amateur Championships three times in four years, winning impressively in 1896 and 1898 but squandering a five hole advantage to lose at the 37th in 1899.

No doubt, Peter Guthrie, the golfer, would stand on the tee watching his partner drive off and wonder just what it was in the golfer's technique which produced a long drive. But Peter Guthrie, the physicist, would contemplate the scientific reasons. Many factors, it seemed to him, might influence the flight, from the atmospheric conditions to the strength of the golfer, from the nature of the material from which the golf-ball was made (then gutta percha) to the angle at which it was dispatched from the tee. But the overriding factor was the back-spin imparted by the club as it struck the ball.

There is so much coverage of ball games on television these days that the spinning, swerving ball, whether it be in soccer, in baseball, in tennis or in table tennis is a commonplace. In fact, the link between spin and trajectory was first noticed by Newton, who in a letter of 1671 to Oldenburg on the subject of the dispersion of light, observed indirectly that differences in air pressure cause balls to have a more complex trajectory than a simple parabolic curve. He recalled that he *'had often seen a tennis ball struck with an oblique racket describe such a curve line'.*

Some further understanding was gained by Benjamin Robins (1707-1751) in the early 1740s. He conducted a series of experiments with curved-barelled muskets and observed that the musket-ball swerved in the opposite direction to the curvature of the barrel. Though he correctly attributed this deviation to spin, he was unable to make predictions for the want of accurately determined coefficients. Heinrich Gustav Magnus (1810-1870), whose name is now attached to this effect, carried out his experiments in a wind tunnel in the middle of the following century. A mathematical explanation was offered by Lord Rayleigh (1842-1919) in a paper of 1877 *on 'The Irregular Flight of a Tennis Ball'*. It highlighted the difference in pressure at the *'front'* and at the *'back'* of the ball.

Meanwhile, between 1865 and 1880, Francis Bashforth (1819-1912), Professor of Applied Mathematics at Woolwich, had carried out a range of experiments on both spherical ballistics and ballistics with elongated heads of various shapes. By 1881 he had made available tables of coefficients of air resistance for velocities between 100 and 2800 ft s^{-1}. Such was the position when Tait's recreational interest in golf spilt over into his scientific life.

Tait first claimed that spin induces a golf ball to deviate from its still-air trajectory in the *'Unwritten Chapter on Golf'*, an article which appeared in *The Scotsman* newspaper in August 1887 and was reproduced in *Nature* the following month. A characteristic of the back-spinning ball is its upward concavity despite the effect of gravity. The ball soars even though its trajectory in the first moments of flight is relatively low. Asserting that this is so is of course a far cry from demonstrating it mathematically.

Tait began sagaciously by exploring the simpler no-spin trajectory. Indeed, by July 1890, he was reporting to the Royal Society of Edinburgh how he had modelled the trajectory of a golf ball using an equation from his first book, *Dynamics of a Particle*:

$$y = \left(\tan\alpha + \frac{ga}{2V_0^2} \right) x - \frac{ga^2}{4V_0^2} \left(e^{2x/a} - 1 \right),$$

where V_0 = initial velocity, in ft s^{-1}, α = initial inclination, in radians, a = fixed ratio, in feet, of the square of the velocity to the deceleration caused by air-resistance and g = acceleration due to gravity, in ft s^{-2}. From experiment and experience he tentatively suggested estimates for the parameters, arriving at the curve:

$$y = 0.258\,x - 2.524\left(e^{x/140} - 1\right)$$

This trajectory attains a maximum height of 62 ft, 372 ft from the tee, but has a range of just 190 yards.

In 'Some Points in the Physics of Golf, II', published in *Nature* in September 1891, Tait again stressed the importance of imparting back-spin to maximise carry. This is effected best by employing a tee so that the lower part of the ball is struck and by roughening the ball's surface. He was unable to explain the uneven surface's drag reducing effect now attributed to turbulence and did not live to witness the advent of the dimpled ball. The no-spin model was clearly inadequate. Even cursory checks out on the golf course bore out the fact that the ball was in the air longer and the range was greater than predicted. A complete model, taking the effect of spin into consideration, was clearly needed. However, there had already been an outcry from members of the Royal and Ancient who considered as an affront any suggestion that they imparted spin to the ball to gain extra distance.

If Tait was initially persuaded to study the aerodynamics of golf balls in order to predict a furthest possible drive then he certainly suppressed this objective, perhaps because it soon became clear that in theory no limiting range appeared to exist, except through human limitations. Instead he fixed the range at 180 yards or even 165 yards and the duration of flight at about 6 seconds and looked at the effect on the other unknowns. He concentrated on validating and refining his model using all the information at his disposal including that from Robins, Magnus and Bashforth.

Tait wrote up his findings in two major papers each entitled 'On the Path of a Rotating Spherical Projectile'. They appeared in the 1893 and 1896 *Transactions of the Royal Society of Edinburgh*, though the publication of the second was delayed until October 1898 while Tait tried to furnish more accurate estimates of the model's parameters through experimentation. For this he drafted in Freddie and other promising young golfers, who in dangerously enclosed laboratory conditions hit balls onto a pad of clay in order to establish a realistic estimate of the initial velocity.

Tait's preferred refinement of the model gives the equation of the trajectory as:

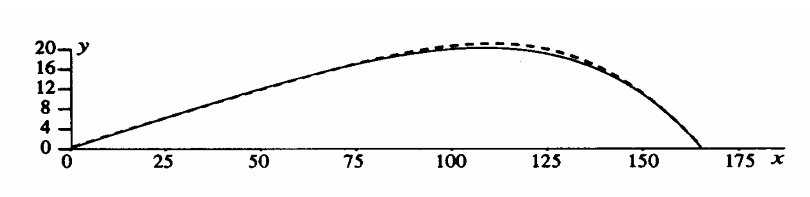
$$y = \alpha x + \frac{ka^2}{V_0}\left(e^{x/a} - 1 - \frac{x}{a}\right) - \frac{ga^2}{4V_0^2}\left(e^{2x/a} - 1 - \frac{2x}{a}\right)$$

where k is a constant of proportionality between lateral acceleration and velocity and the other parameters are as before. The model is clearly complex and is based on a number of physical assumptions, among them that (i) the drag or resistance force on a ball varies nearly as the square of the speed (ii) the lift is proportional to the ball's forward and rotational velocities (iii) the equatorial speed of rotation is constant. Tait consulted the fluid dynamicist, Sir George Stokes, to check how reasonable were these assumptions and received reassurances, although it is true to say that both Tait and Stokes underestimated the speed of rotation. Trajectories vary according to the values of the parameters taken, a variety of them being appended by Tait to his 1893 paper.

Now in order to test Tait's trajectory it is necessary to set various parameters. Tait had what might be described as a preferred typical trajectory with initial velocity 240 ft s^{-1} and initial inclination somewhere between 13.5° and 14°. He took *a* to be 360 ft and set the range at 180 yd. If we now take the initial inclination to be the mid-point of Tait's range, 13.75° or 0.24 radians, *k* is 0.151. We now have a simple trajectory near to that favoured by Tait in his 1896 paper.

$$y = 0.24x + 81.54\left(e^{x/360} - 1 - \frac{x}{360}\right) - 18.1\left(e^{x/180} - 1 - \frac{x}{180}\right)$$

Although Tait observed upward concavity in the early stages of a good drive this phase is completed in the first fifth of a second or so of the flight. This is just one of the conclusions reached by Chris Denley of British Aerospace who has simulated the trajectory on the basis of Tait's assumptions and the modern-day lift theory of Hoerner. In the graph below, scaled in metres rather than yards, Denley's trajectory (broken line) is virtually identical to that of Tait (full line), though the time of flight on the computer model is 5.2 s whereas Tait suggests at least 6 s. Towards the century's end the golfing press teased Tait by spinning a yarn about his son, Freddie, unleashing so mighty a drive that the model was rendered useless. It was almost certainly apocryphal.[6]



We can conclude that Tait's attempt to model the flight of a golf ball is a very impressive piece of work. The calculations required to run it and to validate it must have been desperately lengthy and tedious, and though Tait was ably assisted in this respect by his laboratory student, James Wood, this is still far removed from having access to a modern computer. Tait's development of the trajectory from a gravity-free environment, with lift and drag only, to a realistic vertical plane model with gravitational effects, validated as extensively as existing information and personal observation would allow, shows him to have been a noteworthy mathematical modeller.

Quaternions

Tait bought a copy of Hamilton's 700 page *Lectures on Quaternions* in 1853 and digested the first six chapters without difficulty. Hamilton had realised that a vector *u* can be converted into a vector *v* using one number to adjust the magnitude and three to effect the rotation. The general form of a quaternion *q* is:

$$q = a + ib + jc + kd$$

where *a*, *b*, *c* and *d* are real numbers and *i*, *j* and *k* are vectors with magnitude equal to the square root of -1 in the *x*, *y* and *z* directions. Tait initially struggled to come to terms with the section on applications but he did recognise the possibility of applying the new methods to potential theory. By engaging in an intensive period of correspondence with Hamilton and simultaneously extending the applications of quaternions in original papers, Tait, by the time he arrived in Edinburgh, knew as much about quaternions as did Hamilton.

As Tait and many others had discovered, Hamilton's *Lectures* was not the easiest of reads and former colleagues of Tait's at Cambridge began to call for a more elementary text for use with undergraduates. A minor misunderstanding with Hamilton over the timing of publication led to some delay but eventually in 1867 Tait's *Elementary Treatise on Quaternions* went to press. It was a second book, however, *Introduction to Quaternions*, written in the main by Kelland with advice from Tait, which proved far more accessible to undergraduates.

Tait became increasingly aware of a number of serious deficiencies in the treatment of quaternions to date. It seemed to him that further development would only come by throwing off the shackles of the co-ordinate geometry in which they were cast. So, in 1868, he recast some formulas of Cayley in quaternion form and elicited results much more directly. Lauded by Maxwell for its power this paper, at least in part, landed Tait the Royal Society of Edinburgh's Keith Prize for the years 1867-69 ~ but it also landed him in hot water with Cayley.[7]

Although Maxwell would later distance himself from quaternionists, he was broadly sympathetic at this stage. By November 1870, as Maxwell indicated to Tait, he had decided to 'leaven' his forthcoming *Treatise on Electricity and Magnetism* with quaternions but without casting them in Hamilton's inaccessible form. In fact he would use both the co-ordinates associated with Cayley and the quaternions championed by Tait, arguing that while the two fought for supremacy what he called the 'bilingual' method should be adopted for scientific works: 'ploughing with an ox and an ass together'.

Arthur Cayley, the Sadleirian Professor of Pure Mathematics in Cambridge, took a very different view. Quaternions were like pocket maps, concise and compact, but not as easy to read as full-scale coordinate maps. The matter was a long time coming to a head, but come to a head it did. Writing to Cayley in 1888 Tait argued that *'no problem or subject is a fit one for the introduction of Quaternions if it necessitates the introduction of Cartesian Machinery'*. He described those parts of mathematics which do not lend themselves to quaternionic treatment as *'disaffected or lob-sided'*, no doubt valuable but 'like the occipital ribs and the anencephalous heads in an anatomical museum'. Two years later he called them 'elegant trifles'.

Then in 1894 Tait, rather tactlessly suggested that the notion of a matrix, Cayley's greatest legacy to pure mathematics, is to be found in Hamilton's works. Hardly surprising then that the two remained 'poles asunder' in the words of Tait and their attitudes to each other had somewhat of an edge. In the end, they agreed that each should make out their respective cases for quaternions and co-ordinate geometry and present them for publication side-by-side in *the Proceedings of the Royal Society of Edinburgh*, and this they duly did only months before Cayley died.

Also of interest is the disparity between the views of Tait and the American, Josiah Willard Gibbs (1839-1903), in the early years of the last decade of the century.

A quaternion is part scalar (a) and part vector ($ib + jc + kd$). It seemed to Gibbs that, in the works of Hamilton and his disciple Tait, the scalar part of the quaternion was being used so sparingly that it could be dispensed with altogether to leave just the vector part. He tried out his new algebra on his students at Yale before making it available to a wider audience in two privately published pamphlets in the 1880s. Much the same idea occurred to Oliver Heaviside (1850-1925), who between 1885 and 1887 wrote a number of papers for *The Electrician* happily retaining the scalar and vector products in Hamilton's notation but with no role for quaternions as such.

Tait had become somewhat disillusioned by the lack of progress in spreading the quaternionic gospel. The seeds had been sown by Tait in papers, books and lectures but the stoney ground would not bare wheat. Worse still, shoots of a plant with an apparently simpler structure began to break the ground and it threatened to put the still weak quaternionic plant in the shade. In the preface to the third edition of his *Elementary Treatise* Tait's frustration got the better of him and he called the new vector analysis of Gibbs 'a sort of hermaphrodite monster'!

Gibbs replied with restraint and not a little charm in *Nature* in April 1891: *'If my offence had been solely in the matter of notation, it would have been less accurate to describe my production as a monstrosity, than to characterize its dress as uncouth....'* and went on to argue that vectors are free from artificiality, are superior both in notion and notation and can be extended to four or more dimensions. In his splendid *History of Vector Analysis*[8], Michael Crowe has argued that the paper was carefully constructed and reasonable and it must have appealed greatly to the uncommitted and the neutrals. By contrast, in the debate which followed *'the ratio of heat to light was especially high in the writings of the quaternionists'* who saw Gibbs and Heaviside as upstarts. Though it was vector analysis which eventually held sway, we regard the quaternions of Hamilton and Tait as a precursor, possibly a necessary precursor. Tait's great achievement was that, by recognising that the laws of physics are independent of co-ordinate systems and that mathematical notation should reflect that reality, he was able to recast Hamilton's ideas in a form with which physicists could work

Tait will be remembered as one of the founders of the mathematical theory of knots, as a pioneer researcher into the physics of flight and as the scientist who, in the later decades of the nineteenth century, did more than any other to help free physics from points and axes of reference. These legacies are sufficient in themselves to earn Tait a place high on the physics roll of honour for this period but his collaboration with Thomson and, above all, his friendship and scientific correspondence with Maxwell afford him a unique niche.

Notes

1. This is an abridged and edited version of 'Peter Guthrie Tait, Victorian Scientist', a paper given at the symposium on *Maxwell and His Circle*, organised by the James Clerk Maxwell Foundation in conjunction with the Edinburgh International Science Festival in April 1995 and repeated at the symposium on *Scotland's Mathematical Heritage: Napier to Clerk Maxwell* at the Royal Society of Edinburgh in July 1995. The author is grateful to David Forfar for his helpful comments.
2. Fleeming Jenkin, FRS, Professor of Engineering at Edinburgh University, 1868-1885.
3. James David Forbes, FRS (1809-1868), Professor of Natural Philosophy at Edinburgh University.
4. Listing, J. B. *Vorstudien zur Topologie*, Abgedruckt aus der Göttingen studien, 1847; Göttingen, 1847.
5. Listing, J. B. Der Census räumlicher Complexe, oder Verallgemeinerung des Euler'schen Satzes von Polydern ...... Aus dem 10.bde der Abhandlungen der Königlichen gesellschaft der wissenschaften zu Göttingen; Göttingen, 1862.
6. Denley, C. & Pritchard, C. (1993) 'The golf ball aerodynamics of Peter Guthrie Tait', *Mathematical Gazette* **77**, 298-313.
7. There can be no hiding the fact that Tait had a number of disagreements with other scientists, perhaps as many as eight, and that some were heated.
8. Crowe, M. J. *History of Vector Analysis*, University of Notre Dame Press, 1967; Dover, 1985.