[The following Smith's Prize Exam was taken by James Clerk Maxwell at Cambridge. Question 8 is Stokes' Theorem. (Stokes was a personal friend of Maxwell.) Maxwell completed the exam tied for first.]

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BY GEORGE GABRIEL STOKES, ESQ. M. A.

Lucasian Professor.

1. STRAIGHT lines *AP*, *BP* pass through the fixed points *A*, *B*, and are always equally inclined to a fixed line; shew that the locus of P is a hyperbola, and find its asymptotes.

2. A number of equal vessels communicate successively with each other by small pipes, the last vessel opening into the air. The vessels being at first filled with air, a gas is gently forced at a uniform rate into the first; find the quantity of air remaining in the n^{th} vessel at the end of a given time, supposing the gas and air in each vessel at a given instant to be uniformly mixed.

3. Separate the roots of the equation

$$2x^3 - 9x^2 + 12x - 4.4 = 0,$$

and find the middle root to four places of decimals by Horner's method, or by some other.

4. Investigate a formula in Finite Differences for transforming a series the terms of which (at least after a certain number) are alternately positive and negative, and decrease slowly, into one which is generally much more rapidly convergent.

Example. Find the sum of the series

$$1.4142 - .7071 + .5303 - .4419 + .3867 - .3480 + .3190 - .2962 + \dots$$

5. Given the centre and two points of an ellipse, and the length of the major axis, find its direction by a geometrical construction.

6. Integrate the differential equation

$$(a^2 - x^2)dy^2 + 2xydydx + (a^2 - y^2)dx^2 = 0$$

Has it a singular solution?

7. In a double system of curves of double curvature, a tangent is always drawn at the variable point *P*; shew that, as *P* moves away from an arbitrary fixed point *Q*, it must

begin to move along a generating line of an elliptic cone having Q for vertex in order that consecutive tangents may ultimately intersect, but that the conditions of the problem may be impossible.

8. If X, Y, Z be functions of the rectangular co-ordinates x, y, z, dS an element of any limited surface, l, m, n the cosines of the inclinations of the normal at dS to the axes, ds an element of the bounding line, shew that

$$\iint\left\{l\left(\frac{dZ}{dy}-\frac{dY}{dz}\right)+m\left(\frac{dX}{dz}-\frac{dZ}{dx}\right)+n\left(\frac{dY}{dx}-\frac{dX}{dy}\right)\right\}dS$$
$$=\int\left(X\frac{dx}{ds}+Y\frac{dy}{ds}+Z\frac{dz}{ds}\right)ds,$$

the differential coefficients of X, Y, Z being partial, and the single integral being taken all round the perimeter of the surface.

9. Explain the geometrical relation between the curves, referred to the rectangular co-ordinates x, y, z, whose differential equations are

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R},$$

and the family of surfaces represented by the partial differential equation

$$P\frac{dz}{dx}+Q\frac{dz}{dy}=R.$$

10. Write a short dissertation on the theoretical measure of mass. By what experiments did Newton prove that masses may be measured by their weights? Independently of such experiments, how may it be inferred from the observed motions of the heavenly bodies that the mutual gravitation of two bodies depends only on their masses, and not on their nature? In what two different senses is the term *weight* used?

11. What are the conditions to be satisfied in order that two moving systems may be dynamically as well as geometrically similar?

If it be desired to investigate the resistance to a canal boat moving 6, 8, 10 miles an hour by experiments made with a small model of the boat and canal, if the boat be 36 feet long and its model only 2 ft. 3 in., what velocities must be given to the latter.

12. A rod is suspended at two given points by unequal light elastic strings, in such a manner that the rod is horizontal and the strings are vertical in the position of equilibrium; the rod being slightly disturbed in a vertical plane, in such a manner that no displacement of velocity is communicated to the centre of gravity in a horizontal direction, it is required to determine the motion.

13. Shew how to determine the time of rotation of the Sun about his own axis, and the position of his equator.

14. Rays coming from a luminous point situated in the axis of a large convex lens, and beyond the principal focus, are received after transmission through the lens on a screen field perpendicular to the axis, which is moved from a little beyond the extremity of the caustic surface to a little beyond the geometrical focus; compare, according to the principles of geometrical optics, the illumination at different points of the screen, and at different points of the screen, and at different distances of the screen from the lens, the lengths of any lines in the figure being regarded as known.

Give a sketch of the method of finding the illumination in the neighbourhood of a caustic according to the theory of undulations. What is the general character of the result, and in what natural phenomenon is it exhibited?

15. A glass plate, the surface of which is wetted, is placed vertically in water; shew that the elevation of the fluid varies as the sine of half the inclination of the surface to the horizon, and compare its greatest value with the elevation in a capillary tube of given diameter. Find also the equation of the surface.

16. Explain the different modes of determining the Mass of the Moon.

17. Plane polarized light is transmitted, in a direction parallel to the axis of the crystal, across a thick plate of quartz cut perpendicular to the axis, and the emergent light, limited by a screen with a slit, is analyzed by a Nicol's prism combined with an ordinary prism; describe the appearance presented as the Nicol's prism is turned round, and from the phenomena deduce the nature of the action of quartz on polarized light propagated in the direction of the axis.